A PID Adaptive Sliding Mode Control For An Inverted Pendulum System For An External Disturbance Using Grey Wolf Optimization Technique

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Abstract: Sliding Mode Control is an effective robust control strategy to tackle system uncertainties and also external disturbances. This control strategy has been successfully applied to various practical engineering systems. This paper presents the Proportional-Integral-Derivative Adaptive Sliding Mode Control for an inverted pendulum for an external disturbance. The sliding surface with a Proportional-Integral-Derivative form is taken where the parameter values of the Proportional-Integral-Derivative controller are tuned by using Grey Wolf Optimization algorithm. The control law is designed based on the Lyapunov stability condition. In order to illustrate the efficacy of the proposed design method, a comparison between the Proportional-Integral-Derivative Sliding Mode Control and the proposed Proportional-Integral-Derivative Adaptive Sliding Mode Control using Grey Wolf Optimization is made.

Keywords: Adaptive Sliding Mode Control, Grey Wolf Optimization, Inverted Pendulum, Lyapunov Stability.

I. Introduction

The Inverted Pendulum (IP) system offers a challenging example for control engineers to verify modern control methods. This system is highly nonlinear and unstable one. So the standard linear techniques cannot model the non-linear dynamics of this system. On simulating the inverted pendulum system, the mass mounted pendulum falls down quickly. The inverted pendulum has the property of being unstable and non-linear, multi variable and is highly coupled, that can be considered as a non-linear problem [1]. Due to this characteristic of the inverted pendulum, it makes identification and control of the system more challenging. The controlling of Inverted Pendulum system can be classified into three sections, they are: swing-up control, stabilization, and tracking control. The problem of inverted pendulum is to balance a pole hinged on a movable platform that moves in both the directions, viz. left and right. Common control approaches for stabilization and tracking control of inverted pendulum system like Linear Quadratic Regulator (LQR), Proportional-

Integral-Derivative (PID), Sliding Mode Control (SMC) [2]-[3], Fuzzy Logic Control (FLC),Adaptive Fuzzy Logic Sliding Mode Controller [4] and so forth require a good knowledge of the system and precise tuning in order to obtain desired performances.

In this paper SMC technique has been used for tracking control of the IP system. Generally, the SMC requires asuitable control law for the sliding mode to be reachable in a finite time. The trajectory of the system moves nearer to the sliding surface and stays on the surface. The traditional SMC uses a control law having large control gains which yields an undesired chattering effect. To eliminate the chattering, the boundary layer technique is usually adopted, and many adaptation approaches have been developed to tune the controller gains [5]-[7].

In this correspondence, a systematic and simple Adaptive Sliding Mode Control (ASMC)using Grey Wolf Optimization for tuning the controller parameters is proposed. Based on the Lyapunov theory, the trajectory of the system is guaranteed to move nearer to the sliding surface. Also, the tracking performance is ensured. The simulation results are demonstrated to show its robustness and satisfactory performance.

This paper is organised as below. Section 2 presents the Problem Formulation. Section 3 presents the Grey Wolf Optimization. Inverted Pendulum is briefed up in section 4. In section 5, Designing of Controller is presented and lastly the Conclusion is presented in Section 6.

II. Problem Formulation

The dynamic system that is studied in this paper is governed by the state model [8] is given by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x,t) + g(x,t)u(t) + d(t) \end{aligned} \tag{1}$$

where $x=[x_1, x_2]$ is the state vector, u is the control input, $f(x, t)$ and $g(x, t)$ are well-known nonlinear functions and $d(t)$ is the unknown external disturbances.

III. Grey Wolf Optimization

Grey Wolf Optimizer (GWO) is an optimization technique which is a population-based meta-heuristics algorithm inspired by the grey wolves dominant leadership hierarchy and their hunting mechanism in nature which was proposed by Mirjalili et al. in 2014 [9]. They are considered as the apex predators and are at the peak of the food chain. They basically prefer living in groups (packs), each group contain 5-12 members on average. All the members in the group have a very meticulous social dominant hierarchy as below:

The first level is called Alpha (α). The alpha wolves are considered as the leader of the whole group and they are male and a female. They take responsibilities in decision making, hunting and so on. The pack members have to determine the alpha decisions and they hold their tails down in order to acknowledge the alpha. The alpha wolf is considered as the commanding wolf in the group and the pack members should follow his/her orders.

The second level is called the Beta (β). The betas which are considered as the subordinate wolveshelp in decision making of the alphas. They can either be male or female and when the alpha passes away and becomes old they are considered as the best contestant to be the alpha. The betas strengthen the alpha’s command throughout the group and give feedback to the alpha.

The third level is called Delta (δ). The delta wolves are called the subordinates but they are not the alpha or beta wolves. Though they govern the omega (ω) which is the lowest level in the social hierarchy, they have to submit decisions to the alpha and the beta.

The fourth (lowest) level is called the Omega (ω). The omega wolves are considered as the whipping boy or scapegoat in the group and have to submit to all other dominant wolves. Though they seem unimportant individuals, they are the last wolves allowed to eat. The whole packs are fighting in case of losing the omega.

The dominant social hierarchy of the Grey Wolves are shown in Fig. 1

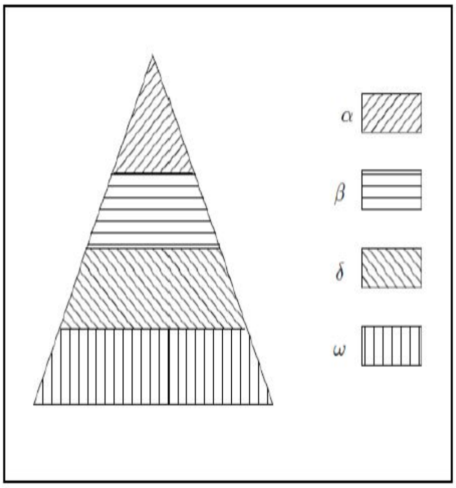

Fig. 1 Social hierarchy of Grey wolf

The Grey Wolf Algorithm is presented as below:

a) *Set the initial values of the population size n, parameter a, coefficient vectors A, C and the maximum number of iterations* Max_{itr}
b) *Set t=0, {Counter initialization}.*
c) *for (i=1:i<n) do*
d) *Generate an initial population* $X_i(t)$ *randomly.*
e) *Repeat*
f) *for (i=1:i<n) do*
g) *Update each search agent in the population*
h) *Decrease the parameter 'a' from 2 to 0.*
i) *Update the coefficients A and C.*
j) *Evaluate the fitness function of each search agent (vector)* $f(X_i)$.
k) *end for*
l) *Update the vectors* X_α, X_β, *and* X_δ.
m) *Set t=t+1 {Iteration counter increasing}*
n) *until* ($t<Max_{itr}$). *{Termination criteria satisfied}*
o) *Produce the best solution* X_α .
p) *Evaluate the fitness function of each search agent (solution)* $f(X_i)$.
q) *end for*
r) *Assign the values of the first, second and the third best solution* X_α, X_β, *and* X_δ. *respectively.*

IV. Inverted Pendulum

An inverted pendulum is a nonlinear system with its centre of mass above its central point. It is often implemented having the centralpoint mounted on a cart that moves horizontally and due to this it is sometimes called as a cart and pole system. Basically, a normal pendulum is stable if it is hanged downwards whereas an inverted pendulum is intrinsically unstable and so to remain upright, it must be balanced. This balancing of the pendulum can be done in many ways such as application of a torque at the pivot point, movement of the pivot point horizontally as a part of feedback system, change in the rotation of the mass mounted pendulum on an axis parallel to the central axis and thus generating a resultant torque on the pendulum, and also by vertically oscillating the pivot point. The physical model of the cart-pendulum system is shown in Fig. 2:

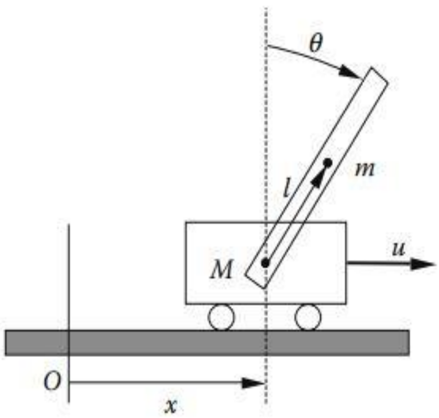

Fig. 2 Cart-pendulum system

Here, M and m denotes the mass of the cart and the inverted pendulum respectively. l denotes the length from the link's centre of gravity to its attachment point. The coordinate x represents the position of the cart and θ is the rotational angle of the pendulum.

Equation (2) describes the dynamical non-linear model of the inverted pendulum system [8]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g(m_p + m_c)\sin(x_1) - m_p L x_2^2 \cos(x_1)\sin(x_1) / L((4/3)(m_p + m_c) - m_p \cos^2(x_1)) \quad (2)$$
$$+ (\cos(x_1) / L((4/3)(m_p + m_c) - m_p \cos^2(x_1))u(t) + d(t)$$

where $x_1 = \theta$, the pole's angle with respect to the vertical axis, $x_2 = \dot{\theta}$, the angle velocity of the pole. m_p and m_c are the mass of the inverted pendulum and the cart respectively. l is the length from the link's centre of gravity to its attachment point.

V. Designing Of Controller

A. Sliding Mode Control (SMC) Design

Sliding mode controller has been introduced in the early of 1960's whose fundamental concept was extracted from the variable structure control [10] which was developed in Russia. The most important stage in the establishment of the SMC control is the structure of sliding surface which is anticipated to be the response to the desired control criterion. The control signal that is reached to the sliding surface is expected to stay on the surface and slide to the origin which is the desired position as shown in the Fig. 3 [10].

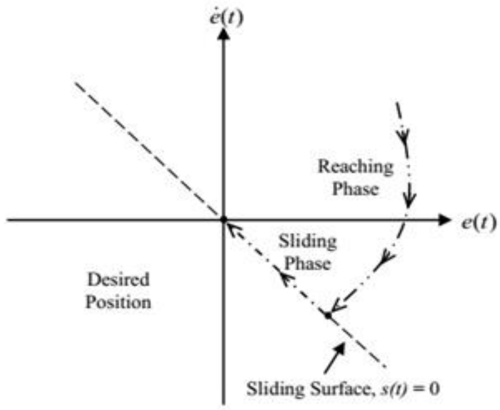

Fig. 3 The structure of the sliding mode control

In general, the sliding surface of the SMC control [10] can be obtained according to (3)

$$(\lambda + d/dt)^{n-1} e(t) \tag{3}$$

where n denotes the system model order to be controlled.
In this paper, a PID sliding surface [10] is used and is given by (4):

$$s(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau + k_d \dot{e}(t) \tag{4}$$

where k_p, k_i and k_d are referred to the PID parameters.
The difference between the actual position and the desired trajectory yields the tracking error and is expressed in (5) given by:

$$e = x_{1d} - x_1 \tag{5}$$

The control law of the sliding mode control consists of switching control and equivalent control. The first one is the equivalent control u_{eq} corresponding to sliding phase when $\dot{s}(t) = 0$ and the second one is the switching control u_{sw} corresponding to the reaching phase $s(t) \neq 0$. The tracking error will then reached to the sliding surface and converge to the equilibrium point where $s(t) = \dot{s}(t) = 0$, where the derivative of the sliding surface is represented as:

$$\dot{s}(t) = k_p \dot{e}(t) + k_i e(t) + k_d \ddot{e}(t) \tag{6}$$

The control law is adopted as:

$$u = (k_p(\dot{x}_{1d} - \dot{x}_1) + k_i(x_{1d} - x_1) + k_d(\ddot{x}_{1d} - f(x) - dt) + \eta \,\mathrm{sgn}(s)/(g(x)k_d)) \tag{7}$$

The Lyapunov function is a scalar function of the system state variables V(x)>0. The control law decreases this function through time, i.e. V(x) <0.

The Lyapunov function is defined as:

$$V = (1/2)s^2 \tag{8}$$

And its time derivative is given as:

$$\dot{V} = s\dot{s} \tag{9}$$

where $\dot{s}$ is given by:

$$\dot{s} = k_p\dot{e} + k_i e + k_d(\ddot{x}_{1d} - f(x,t) - g(x,t)u(t) - d(t)) \tag{10}$$

Now, substituting (7) in (10), we get:

$$\dot{s} = k_p\dot{e} + k_i e + k_d(\ddot{x}_{1d} - f(x,t) - (1/k_d)(k_p\dot{e} + k_i e + k_d(\ddot{x}_{1d} - f(x,t) + \eta\,\mathrm{sgn}(s) + ks)) - d(t)) \tag{11}$$

Therefore

$$s\dot{s} = -s(kd(\eta\,\mathrm{sgn}(s) + ks + d(t))) < 0 \tag{12}$$

Thus, V is negative definite and if k is a positive constant, $V(t)$ will tend to zero exponentially with k value.

The block diagram of Sliding Mode Control with GWO technique for an inverted pendulum system is shown in Fig. 4

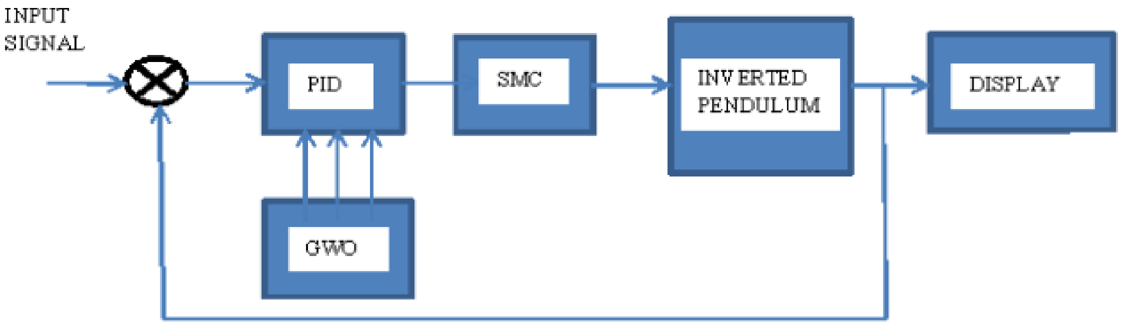

Fig. 4 Block diagram of PID-SMC with GWO

B. Adaptive Sliding Mode Control (ASMC) Design

In this design, a PID sliding surface [10] is used given by (4) and θ_d is the ideal position signal and $e = x_1 - \theta_d$ as the position tracking error.

The Lyapunov function is represented as

$$V = (1/2)g(x_1)s^2 + (1/2\gamma_1)(\phi_1 - \hat{\phi}_1)^2 + (1/2\gamma_2)(\phi_2 - \hat{\phi}_2)^2 + (1/2\gamma_3)(\phi_3 - \hat{\phi}_3)^2 \tag{13}$$

where γ_i>0, $\hat{\phi}_i$ is estimation value of ϕ_i (i=1, 2, 3).

Now choosing $V_1 = (1/2)g(x_1)s^2$ (14)

$$V_2 = (1/2\gamma_1)(\phi_1 - \hat{\phi}_1)^2 + (1/2\gamma_2)(\phi_2 - \hat{\phi}_2)^2 + (1/2\gamma_3)(\phi_3 - \hat{\phi}_3)^2 \tag{15}$$

Then

$$\dot{V}_1 = (1/2)\dot{g}(x_1)s^2 + g(x_1)s\dot{s} \tag{16}$$

$$\dot{V}_2 = -(1/\gamma_1)(\phi_1 - \hat{\phi}_1)\dot{\hat{\phi}}_1 - (1/\gamma_2)(\phi_2 - \hat{\phi}_2)\dot{\hat{\phi}}_2 - (1/\gamma_3)(\phi_3 - \hat{\phi}_3)\dot{\hat{\phi}}_3 \quad (17)$$

where, $g(x_1) = \phi_1 \sec(x_1) - \phi_3 \cos(x_1)$, $\phi_1 = (m_c + m_p)((I + m_p l^2)/(m_p l))$, $\phi_2 = (m_c + m_p)g$ and $\phi_3 = m_p l$

also, $\dot{g}(x_1) = (\phi_1 \sec(x_1)\tan(x_1) + \phi_3 \sin(x_1))x_2$,

$$g(x_1)s\dot{s} = g(x_1)s(k_p\dot{e} + k_i e + k_d \ddot{e}) = s((\phi_1 \sec(x_1) - \phi_3 \cos(x_1))(k_p \dot{e} + k_i e - k_d \ddot{\theta}_d) + k_d(u + \phi_2)\tan(x_1) - \phi_3 x_2^2 \sin(x_1) - d(t))$$

So,

$$\dot{V}_1 = \phi_1((1/2)s^2 x_2 \sec(x_1)\tan(x_1) + s\sec(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d)) + \phi_2(sk_d \tan(x_1)) + \phi_3((1/2)x_2 s^2 \sin(x_1) - s\cos(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d) - k_d s x_2^2 \sin(x_1)) + sk_d(u - d(t)) \quad (18)$$

The control law is defined as:

$$u = -\eta sign(s) + dt - (1/k_d)(\hat{\phi}_1(0.5 s x_2 \sec(x_1)\tan(x_1) + \sec(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}))) + \hat{\phi}_2(k_d \tan(x_1)) + \hat{\phi}_3(0.5 s x_2 \sin(x_1) - \cos(x_1)(k_p \dot{e} + k_i e - \ddot{\theta}) - k_d x_2^2 \sin(x_1))) \quad (19)$$

where η a constant and $\eta \geq$ max|dt| and dt is the external disturbance.
Now, substituting (14) into (13), we get:

$$\dot{V}_1 = -\eta s \operatorname{sgn}(s) k_d - k_d k s^2 + (\phi_1 - \hat{\phi}_1)((1/2)s^2 x_2 \sec(x_1)\tan(x_1) + s\sec(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d)) + (\phi_2 - \hat{\phi}_2)(sk_d \tan(x_1)) + (\phi_3 - \hat{\phi}_3)((1/2)x_2 s^2 \sin(x_1) - s\cos(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d) - k_d s x_2^2 \sin(x_1)) \quad (20)$$

Therefore,

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = -\eta s \operatorname{sgn}(s) - k_d k s^2 + (\phi_1 - \hat{\phi}_1)((1/2)s^2 x_2 \sec(x_1)\tan(x_1) + s\sec(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d)) - (1/\gamma_1)\dot{\hat{\phi}}_1) + (\phi_2 - \hat{\phi}_2)(sk_d \tan(x_1) - (1/\gamma_2)\dot{\hat{\phi}}_2) + (\phi_3 - \hat{\phi}_3)(((1/2)x_2 s^2 \sin(x_1) - s\cos(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta}_d) - k_d s x_2^2 \sin(x_1)) - (1/\gamma_3)\dot{\hat{\phi}}_3) \quad (21)$$

The adaptive law is designed as

$$\dot{\hat{\phi}}_1 = \gamma_1(0.5 s^2 x_2 \sec(x_1)\tan(x_1) + s\sec(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta})) \quad (22)$$

$$\dot{\hat{\phi}}_2 = \gamma_2 s k_d \tan(x_1) \quad (23)$$

$$\dot{\hat{\phi}}_3 = \gamma_3(0.5 s^2 x_2 \sin(x_1) - s k_d x_2^2 \sin(x_1) - s\cos(x_1)(k_p\dot{e} + k_i e - k_d \ddot{\theta})) \quad (24)$$

In practice, the discontinuous term sgn(s) in (7) and (19) may cause phenomenon of chattering in the sliding mode. In order to restrain the chattering effect, a saturated function sat(s) is adopted to smooth the chattering which is given by (25):

$$sat(s)=\begin{cases}1, s>\Delta \\ ks, |s|\le\Delta, k=1/\Delta \\ -1, s<\Delta\end{cases} \tag{25}$$

where ‘ Δ ’ is the boundary layer. But, it is to be noted that too wide boundary layer may give steady error and too narrow boundary layer may not reduce the chattering effect completely. So, the boundary layer width should be chosen according to each particular control system.

The following Fig. 5 shows the block diagram of the Adaptive Sliding Mode Control with GWO technique for an inverted pendulum system.

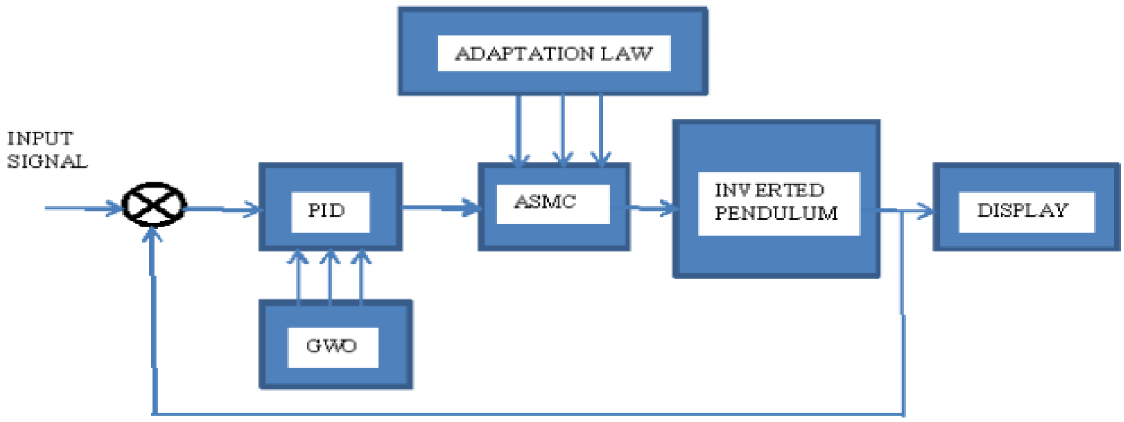

Fig. 5 Block diagram of PID-ASMC with GWO

VI. Simulation And Results

Here, the following parameters of the inverted pendulum are tabulated in Table I as below:

TABLE I. Parameter Values Of Inverted Pendulum

Parameters	Values
m_p	0.1 kg
m_c	1 kg
g	9.8m/s^2
l	0.5 m

Here, the objective is to track the desired state x_{1d} or θ_d which is given by:

$$x_{1d} = 0.3\sin(0.2\pi t) \tag{26}$$

Where the initial conditions (x1(0); x2(0)) are set to (0.4, 0).

And the disturbance against the controllers is:

$$d(t) = 0.1\sin(t) \tag{27}$$

The simulation result with disturbance for sinusoidal input is shown in Fig. 6 below:

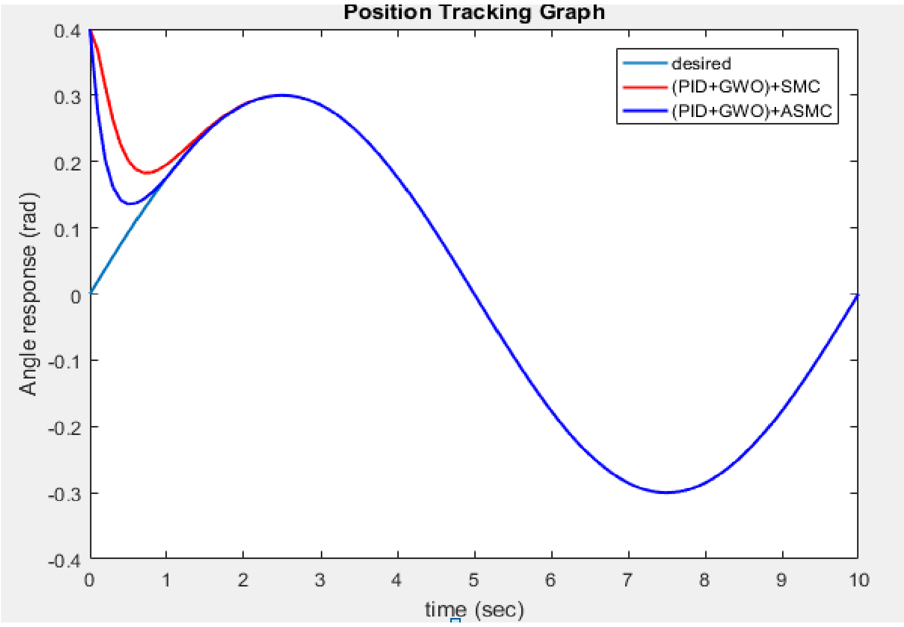

Fig. 6 Angle position with disturbance for sinusoidal input

And the simulation result with disturbance for step input is shown in Fig. 7

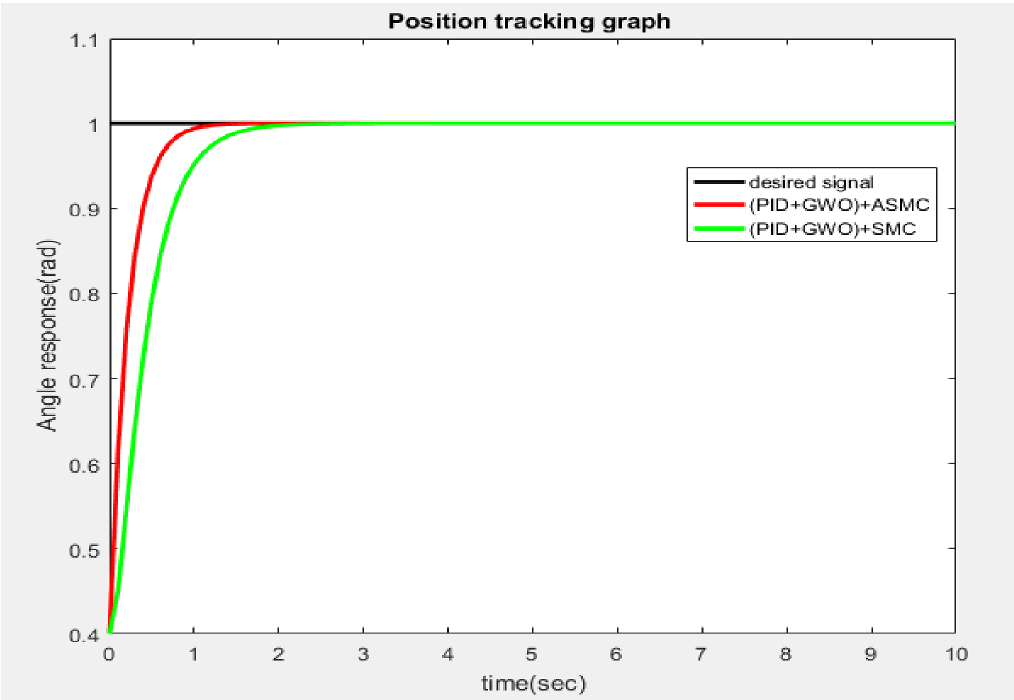

Fig. 7 Angle position with disturbance for step input

Here, MATLAB/SIMULINK environment is used for simulation. By utilizing the GWO algorithm, the optimized parameter values of the controllers are tabulated in Table II and Table III.

TABLE II. OPTIMIZED VALUES FOR SINUSOIDAL INPUT

Parameters	(PID+GWO)+SMC	(PID+GWO)+ASMC
K_p	145.4966	166.7916
K_i	143.6657	128.7977
K_d	124.1084	133.7584

TABLE III. OPTIMIZED VALUES FOR STEP INPUT

Parameters	(PID+GWO)+SMC	(PID+GWO)+ASMC
K_p	188.522	178.2551
K_i	191.3182	122.7811
K_d	108.4997	128.6018

The objective function values by using GWO algorithm applied to both the PID sliding surface of the SMC and ASMC controllers are tabulated in Table IV and Table V respectively.

TABLE IV. OBJECTIVE FUNCTION VALUES FOR SINUSOIDAL INPUT

Controllers	ISE (objective function values)
(PID+GWO)+SMC	0.046853
(PID+GWO)+ASMC	0.019132

TABLE V. OBJECTIVE FUNCTION VALUES FOR STEP INPUT

Controllers	ISE (objective function values)
(PID+GWO)+SMC	0.072786
(PID+GWO)+ASMC	0.04009

VII. CONCLUSION

In dealing with the Inverted Pendulum system which is highly non-linear with uncertain behaviour is in high demand. In this paper, the performance of SMC with PID sliding surface and the proposed ASMC with PID sliding surface using GWO technique has been assessed. The numerical and simulation results indicate that the ASMC with PID sliding surface using GWO shows the better tracking performance than the SMC with PID sliding surface using GWO for the non-linear system.

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